Two-pion exchange potential and the πN amplitude

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Abstract

We discuss the two-pion exchange potential which emerges from a box diagram with one nucleon (the spectator) restricted to its mass shell, and the other nucleon line replaced by a subtracted, covariant πN scattering amplitude which includes Δ, Roper, and D_{13} isobars, as well as contact terms and off-shell (non-pole) dressed nucleon terms. The πN amplitude satisfies chiral symmetry constraints and fits πN data below ~ 700 MeV pion energy. We find that this TPE potential can be well approximated by the exchange of an effective sigma and delta meson, with parameters close to the ones used in one-boson-exchange models that fit NN data below the pion production threshold.

OVERVIEW AND FRAMEWORK

At low energies it seems to be possible to describe hadronic processes in terms of nucleons and mesons alone. Effective dynamical models for the NN interaction can be constructed with these underlying degrees of freedom. Meson exchange models are then favored since they allow a straightforward consistent connection to the electromagnetic and electroweak reactions. However, to extend these models up to a few GeV lab energy, as needed for the study of many reactions produced at CEBAF and other modern accelerators, one must include many inelastic channels associated with the explicit production of pions (and other mesons). These channels are often described by baryon isobars, including their decays and propagation.

At these higher energies, one must also include relativistic effects which arise not only from the kinematics and the need to correctly transform amplitudes from one frame to another, but also from the nonlocallity and energy dependence which characterize relativistic interactions obtained from an effective field theory. The correct treatment of unitarity also requires that the the renormalization of the meson-nucleon vertices and the simultaneous dressing of the "bare" nucleons by meson rescattering be consistent with the meson exchange picture of the nuclear force [1]. It is difficult to deal consistently with this complex

problem within strictly non-relativistic formalisms [2], and we will show in this paper that a relativistic effective field theory provides a natural way to attack this problem.

Using the covariant spectator (or Gross) equations [3], relativistic one boson exchange (OBE) models have been found which give an excellent description of elastic NN scattering (ie. below the pion production threshold) [4]. It is the purpose of this paper to outline how this program should be extended to include the inelastic effects arising from the explicit production of pions in intermediate states. We will limit ourselves to consideration of the lowest lying inelastic cut arising from the πNN channel, and postpone any discussion of how to treat the $n\pi NN$ channels (with $n > 1$).

Consider a simple model with a nucleon (N) and its first excited state (N^*) coupled to a pion. Assume that the two baryons N and N^* are both bound states of some other fundamental constituents confined inside the baryon volume, so that in the absence of the interaction with the pion they are both stable states. The coupling to the pion then dresses the N and, if we assume $m_{_{N^*}} > m_{_N} + \pi$, allows the N^* to decay to $N + \pi$. To treat the πNN cuts consistently in a *time-ordered formalism*, it is then sufficient to ignore diagrams which contribute to $2\pi NN$ cuts, such as those with πNN^* and N^*N^* channels. Examples of classes of diagrams which must be included and classes which can be ignored are shown in Fig. 1. The diagrams in Figs. 1a are iterations of bare N and N^* states, the N^*N^* channel being excluded because its cuts are too high. Diagrams in Figs. 1b are generated from those in 1a through the consistent application of unitarity. They include nucleon dressing, the virtual transition of $N^* \to N$ (when not forbidden by selection rules), and the transitions $N^* \to \pi N \to N^*$, which give the N^* its width in this model. Diagrams in Figs. 1c all have higher cuts and are ignored in this approximation. Note that this eliminates the need to consider the $\pi N^* N^*$ vertex and the πN^* dressing of the baryons.

The time ordered formalism is not covariant, and it is desirable to find diagrams from covariant perturbation theory which include the physics contained in Figs. 1a and b. Motivated by the advantages of the Gross or "spectator" prescription for dealing with the relative energy variable in four-dimensional integral equations [3], we chose the class of diagrams shown in Fig. 2. In these figures the "X" means that the nucleon is on-shell; which means that its propagator is replaced by

$$\frac{\not{p} + m_{_N}}{m_{_N}^2 - p^2} \to \delta_+(m_{_N}^2 - p^2) \sum_s u(p,s)\overline{u}(p,s)\,. \tag{1}$$

Restricting the spectator nucleons in NN^* and $N\{\pi N\}$ loops to their mass-shell does not alter the πNN unitarity cuts contained in these diagrams. This choice is also consistent with our aim to relate the present work to the one of Ref. [4], and differs from the choice we would have made if we wished to extend Refs. [5–8]. In Ref. [5] a non-relativistic equation is used, in Ref. [6] the Bethe-Salpeter equation is solved in ladder approximation, and in Refs. [7,8] the Blankbecler and Sugar reduction is used. Each of these approaches would suggest a particular way of handling the diagrams in Fig. 1. Since the Gross equation specifies that one particle is on the mass-shell in any intermediate state, the covariant class of diagrams shown in Fig. 2, with the substitution of Eq. (1), is the correct way to incorporate the physics of Fig. 1 into that formalism.

Our discussion implies that the full treatment of the coupled $NN - \pi NN$ system requires the summation of the class of diagrams shown, up to 6th order, in Fig. 3. A general discussion

of the coupled equations which sum this infinite class of diagrams will be presented elsewhere. Here we will limit ourselves to an evaluation of the kernel shown in Fig. 4. When this kernel is added to the OBE kernel used in the relativistic treatment of the NN channel below the pion production threshold [4], we have an equation which sums the subclass of diagrams shown in Fig. 3a, but omits contributions of the type illustrated in Fig. 3b. Study of the kernel shown in in Fig. 4 will be a first estimate of the effect of πN rescattering in the spectator formalism.

This diagram is evaluated by carrying out the three-dimensional numerical integration over the internal three-momentum because the spectator prescription of Eq. (1) specifies the internal energy. Furthermore, since the iteration of the one pion exchange (OPE) graphs with bare NN intermediate states are already included when the NN scattering is described using the OBE kernel, the contribution from the undressed direct nucleon pole (with bare Dirac nucleon propagator describing the intermediate nucleon) must be subtracted from the πN amplitude which is used in Fig. 4 in order to avoid double counting. The dressing of the nucleon pole term which arises from πN rescattering must be retained. In a short-hand symbolic notation, the kernel obtained from Fig. 4 is then

$$V_{\rm TPE} = M^4 - \int d^3k \; V_\pi \, \mathcal{G} \, V_\pi \, , \tag{2}$$

where M^4 is a diagram similar to Fig. 4 but with the "blob" replaced by the *full* πN scattering amplitude, V_π is the OPE kernel, and $\mathcal{G}$ the symmetrized sum of one undressed Dirac propagator and one mass-shell projection operator as given in Eq. (1). Subtracting the iterated OBE means that (2) can be added to the OBE kernel of Ref. [4] without double counting.

Before turning to the detailed study of the two pion exchange (TPE) kernel, we briefly address a question of consistency which has posed a serious problem in previous studies of pion production. Consider the diagram shown in Fig. 2b1. In this diagram, one of the nucleons is off-shell and is being dressed by the πN interaction while the other is a physical on-shell nucleon. How can these two nucleons be considered identical when they are treated in such a different way? This question has a very simple answer in the spectator formalism. First, we must symmetrize the interaction, and this is the reason why both diagrams shown in Fig. 4 must be included in the kernel. If, in addition, we require that the *properties of a dressed nucleon be identical to a free nucleon when it is on mass-shell*, then the two nucleons will be identical in every respect. The model we will use to describe the πN scattering part [9] of the TPE kernel has precisely this feature.

THE TPE KERNEL

As discussed above, we limit ourselves in this work to a calculation of the TPE potential given in Eq. (2) and shown diagramatically in Fig. 4, in which the full πN amplitude (with the *direct nucleon pole term removed*) is embedded in the two-nucleon system. As we have emphasized above, it is essential to remove the direct nucleon pole term from the πN scattering amplitude (so the "blob" in Fig. 4 represents this difference) in order to avoid counting the nucleon box contribution more than once.

According to the conventional wisdom underlying the one boson exchange model, the process of Fig. 4 contributes to the intermediate and short range behavior of the NN interaction, and is represented by the exchange of effective bosons with masses heavier than two pion masses (ie. greater than $\simeq 280$ MeV). Such a representation cannot be valid above the πNN threshold, however, where production of a physical πNN intermediate state ensures that the NN scattering will be inelastic [7]. Our calculation will be applied below the pion production threshold, and is only meant to be a first step to estimating the contributions from nucleon resonances and to preparing the way for the extension of the models of Ref. [4] to higher energies. We will conclude from the results that the diagram of Fig. 4 gives a good explanation of the appearance of effective σ and δ mesons, which describe the TPE exchange but do not correspond to real mesons.

The calculation of the diagram shown in Fig. 4 is greatly simplified if we use a πN amplitude with a simple, manifestly covariant sructure. We take here the covariant πN amplitude already constructed in Ref. [9] and applied successfully (with minor changes) to the description of pion photoproduction [10] and to the calculation of the Δ isobar E_2/M_1 ratio. The covariant πN amplitude is decomposed into contributions even and odd under the exchange of isospin

$$T = T^{+}\delta_{ij} + T^{-}[\tau_j \, , \, \tau_i] \tag{3}$$

where each component depends only on the total four-momentum of the πN system, which we denote by P,

$$T^{\pm}(P^2) = A^{\pm}(P^2) + B^{\pm}(P^2)\not{P} \, . \tag{4}$$

We use the latest version of the model given in Ref. [10]. Details can be found both in both [9] and [10], where a description of the phase shifts and inelasticities is also given.

This calculation of the effective TPE potential differs substantially from previous work. Most other calculations have limited contributions to the πN amplitude which appears in Fig. 4 to the Δ isobar [5–7]. Some have used dispersion relations to include other resonances [8]. Our calculation includes the following features

- the πN amplitude is obtained beyond the tree-level, by solving a relativistic two-body scattering integral equation where the pion is considered on its mass-shell in all the intermediate states;

- the kernel of the πN equation includes the nucleon, Δ, Roper and D_{13} poles together with a contact term which includes effects from the crossed nucleon pole and σ and ρ exchange terms, essential to satisfy chiral symmetry constraints and include the physics of the inelastic πNN channel approximately;

- the spin 3/2 particles (Δ and D_{13}) are described by a propagator that includes a covariant spin 3/2 projection operator with the pole at $P^2 = 0$ cancelled by a zero in the baryon form factor;

- the baryon form factors vanish not only at at $P^2 = 0$, but also in the space-like region $P^2 < 0$ (which lies far away from the physical scattering region).

Since the amplitude contains a description of the crossed baryon poles, the crossed-box diagram shown in Fig. 5 is automatically included, although in an approximate fashion.

The overall isospin factors for the isospin 1/2 and 3/2 contributions to the πN amplitudes are ($P^{ij}_{1/2}$ and $P^{ij}_{3/2}$ being the isospin projection operators for the 1/2 and 3/2 cases)

$$
\begin{aligned}
(\tau_j\tau_i)_1 (P^{ij}_{1/2})_2 &= (P^{ij}_{1/2})_1 (\tau_j\tau_i)_2 = \frac{1}{3}(\tau_j\tau_i)_1(\tau_j\tau_i)_2 = 1 - \frac{2}{3}\tau_1\cdot\tau_2 \\
(\tau_j\tau_i)_1 (P^{ij}_{3/2})_2 &= (P^{ij}_{3/2})_1 (\tau_j\tau_i)_2 = (\tau_j\tau_i)_1 \left[\delta_{ij} - (P^{ij}_{1/2})_2\right] = 2 + \frac{2}{3}\tau_1\cdot\tau_2
\end{aligned}
\tag{5}
$$

This very simple argument shows that the strength of the exchange of an effective isoscalar meson is the sum of contributions from both $I = 1/2$ and $I = 3/2$ πN channels (weighted 1 to 2), while the strength of the exchange of an effective isovector meson is the *difference* of the $I = 1/2$ and $I = 3/2$ contributions. In a model dominated by the Δ isobar, we expect the effective strengths of the isoscalar and isovector exchanges to be in the ratio of three to one, and to have the same sign. However, when the $I = 1/2$ contributions to the internal πN scattering are important (which is true for the D_{13} contribution), the relative strengths of the different effective exchanges can be very different. Our calculation shows that the relative sizes of the Δ and D_{13} amplitudes are such that, in the isovector exchange channel, there is an almost perfect (and unexpected) cancellation between the two, leaving a very small (but not zero) effective isovector exchange force. This is probably the most surprising result of this work. Signs of this cancellation were present in Ref. [8], although it was not seen as spectacularly, since in that paper the D_{13} resonance was modeled with a width of roughly 1/4 of its present experimental value. Yet, in that calculation, the D_{13} resonance was already found to be needed to oppose the contribution of the Δ to the short range central part of the NN potential.

RESULTS AND CONCLUSIONS

If all of the external nucleons are on-shell, the TPE exchange amplitude shown in Fig. 4 consists of six independent helicity amplitudes for each possible value of the isospin ($I = 0$ or $I = 1$) in the t (exchange) channel. (For a full discussion of the definition of NN helicity amplitudes see, for example, Ref. [4].) All of these amplitudes were evaluated and compared to those which are obtained from the exchange of a scalar meson (either σ, if $I = 0$, or δ, if $I = 1$) with an arbitrary mass and coupling constant, and a mesonNN form factor with a cutoff mass around 1–1.2 GeV. The form factor mass was held constant during each fit, and was chosen to most closely match the energy dependence of the TPE amplitude. Once the form factor mass was chosen, the meson mass and coupling constant were adjusted to give the best fit.

Figure 6 shows the quality of the fit for several helicity amplitudes at two different relative momenta P of the NN pair, both corresponding to an energy below the one pion production threshold (which is at $P = 370$ MeV/c). Note that the fits are quite good, showing that these TPE potentials are well represented by a sum of σ and δ exchanges (at least below the pion production threshold). For each of the isoscalar and isovector channels we also tried fitting to the sum of scalar plus vector exchange terms, which for the $I = 0$ channel included σ and ω exchanges, and for the $I = 1$ channel included δ and ρ exchanges. We

found that the fits did not significantly improve, showing that the TPE kernel provides little justification for the addition of effective vector mesons to our effective NN force. In fact, the deviation from zero of the differences $M^4_{++-+} - M^4_{+-++}$ and $M^4_{++++} - M^4_{++--}$ cannot be explained through a σ (δ) exchange alone, but these differences, which increase slowly with energy, were found to be very small. This result confirms the observation that the full TPE amplitudes can be approximated by the exchange of scalar mesons only (at least below the pion production threshold).

The effective meson parameters resulting from the fits are shown in Table I. In both the isoscalar and isovector cases the values obtained for the mass and coupling constant are somewhat smaller than the values obtained from OBE models, but are still in reasonable agreement with those results.

Figure 7 shows how a typical NN amplitude (M^4_{++++}) is built up from the different parts of the πN amplitude, and also displays the cancellation of the Δ and the D_{13} contributions in the isovector channel. It also shows that the Δ contribution dominates the isoscalar exchange channel.

We conclude that the large effective σ exchange, together with the comparatively small effective δ exchange, both of which are characteristic features of OBE models, can be largely explained by the TPE contribution, *provided the contribution of the D_{13} resonance is properly included*.

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TABLES

	mass (IIB)	mass (TPE)	$g^2/(4\pi)$ (IIB)	$g^2/(4\pi)$ (TPE)
σ	522	508	4.87	4.02
δ	428	345	0.24	0.20

TABLE I. Comparison of the sigma and delta masses and couplings obtained for Model IIB of Ref. [4] with the same parameters obtained from a fit to the TPE helicity amplitudes. Masses are in MeV; the δ meson was denoted by σ_1 in Ref. [4].

FIGURES

FIG. 1. Time ordered diagrams describing pion production and rescattering. (a) Pion exchange, including resonance contributions; (b) self energy diagrams generated by unitarity from the same cuts shown in (a); (c) diagrams which can be ignored because they have cuts above the $2\pi NN$ production threshold.

FIG. 2. Covariant diagrams for use in the spectator formalism which include the physics of the diagrams shown in Fig. 1a and 1b.

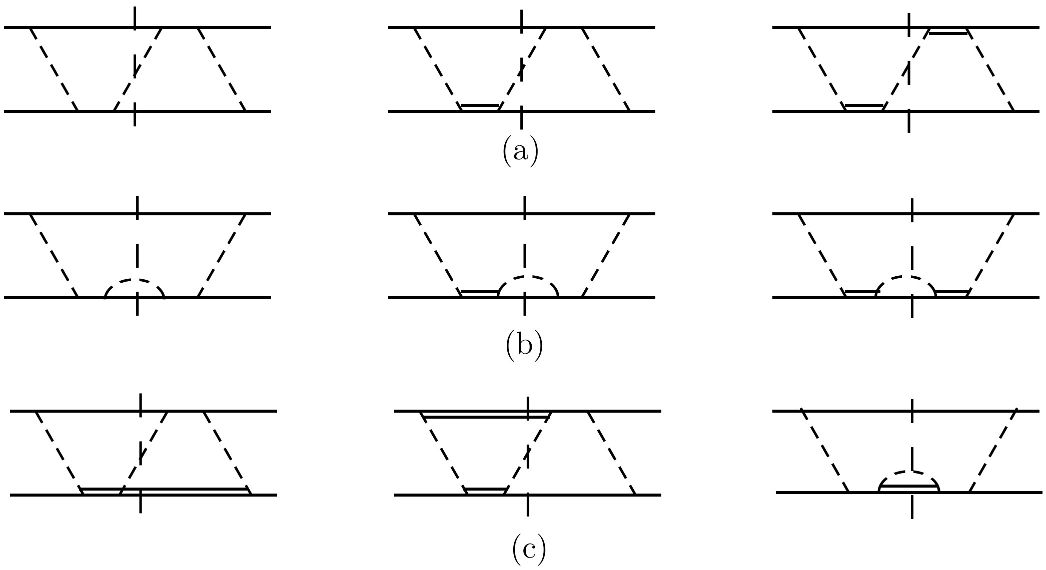
FIG. 3. All diagrams up to 6th order in the πNN coupling which are iterations of the basic interactions included in Fig. 2.

FIG. 4. The effective TPE amplitude studied in this paper. The "blob" is the full πN amplitude with the direct nucleon pole term removed.

FIG. 5. the crossed pion exchange diagram included in the TPE kernel.

FIG. 6. Isoscalar and isovector TPE helicity amplitudes, at relative NN momenta of 60 MeV/c and 240 MeV/c, compared to the fitted sigma ($I = 0$) and delta ($I = 1$) one boson exchange amplitudes.

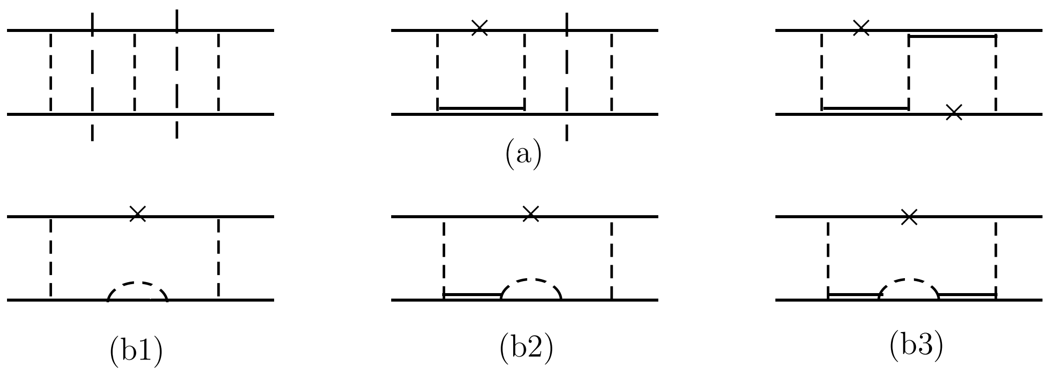
FIG. 7. Contributions of individual parts of the πN amplitudes to the $V_{\rm TPE}^{++++}$ amplitude. The left two pannels are the isoscalar exchange amplitude, the right two are isovector, the top two are for $P = 60$ MeV, and the bottom two for $P = 240$ MeV. Note that the isoscalar amplitudes are well approximated by the Δ contribution alone, while the D_{13} and Δ contributions cancel for the isovector terms.



(a)

(b)

(c)


(a)
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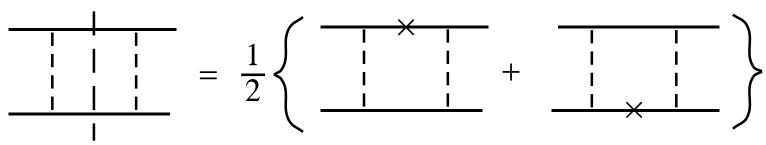
(b2)
(b3)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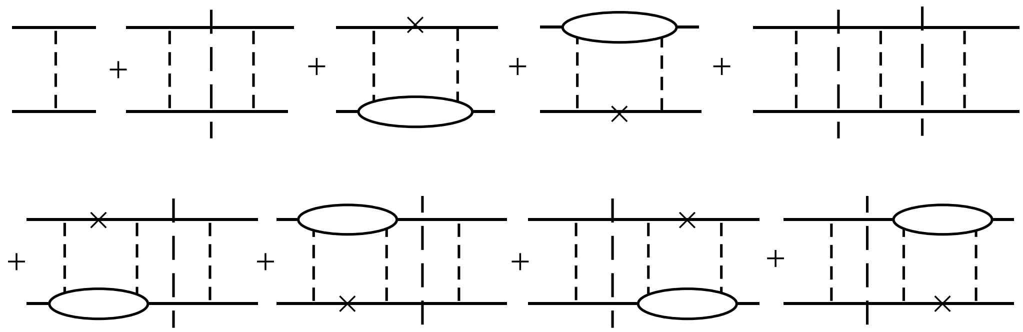
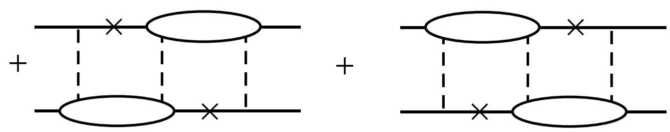


(a)



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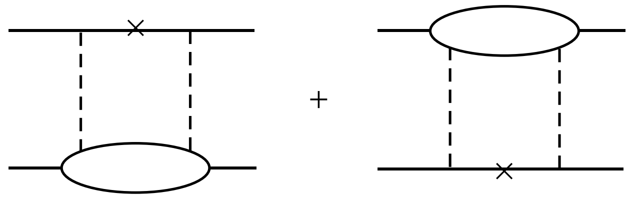
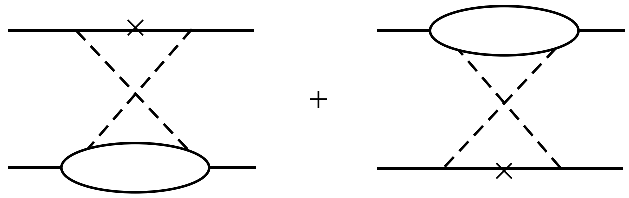
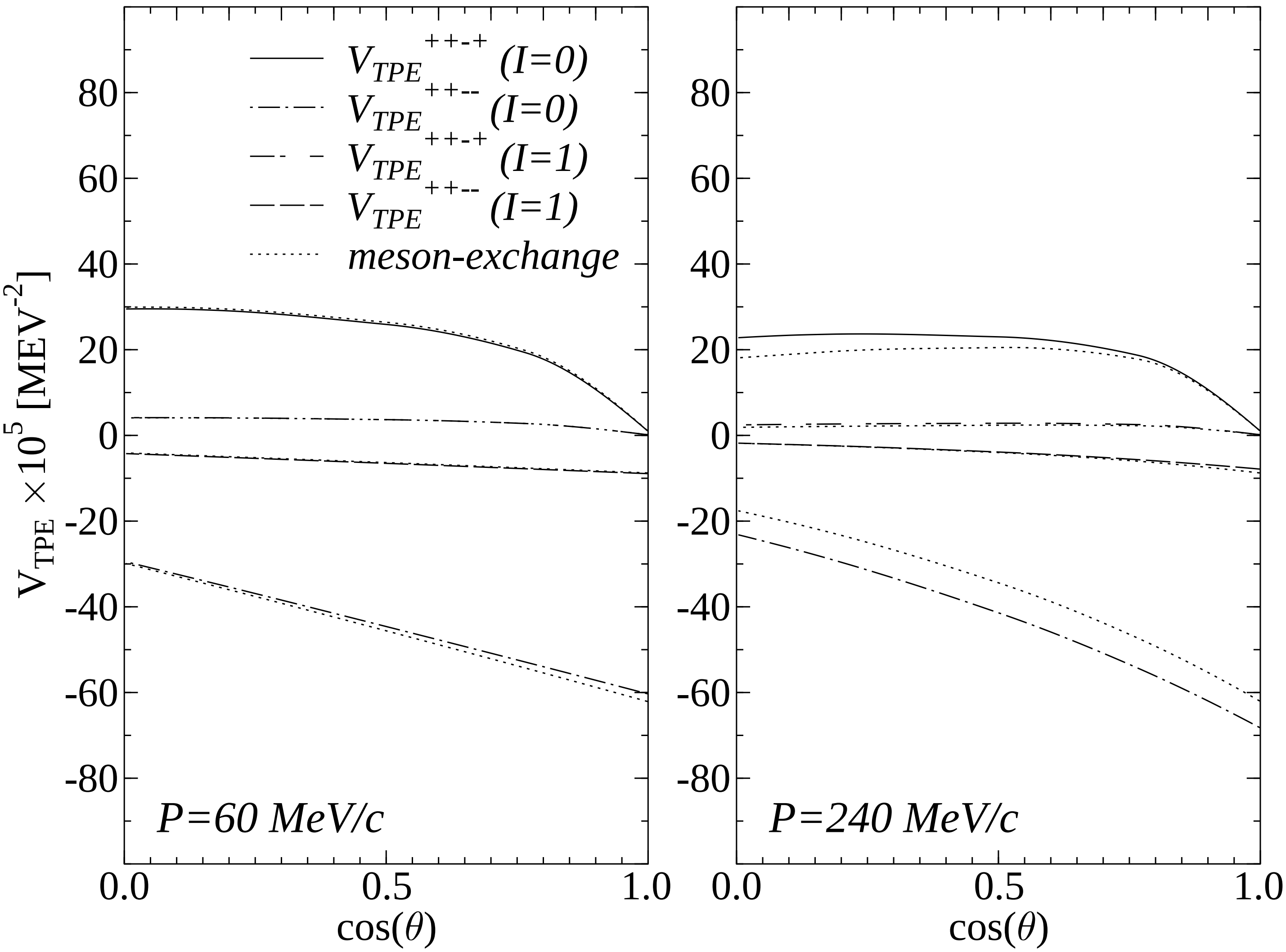
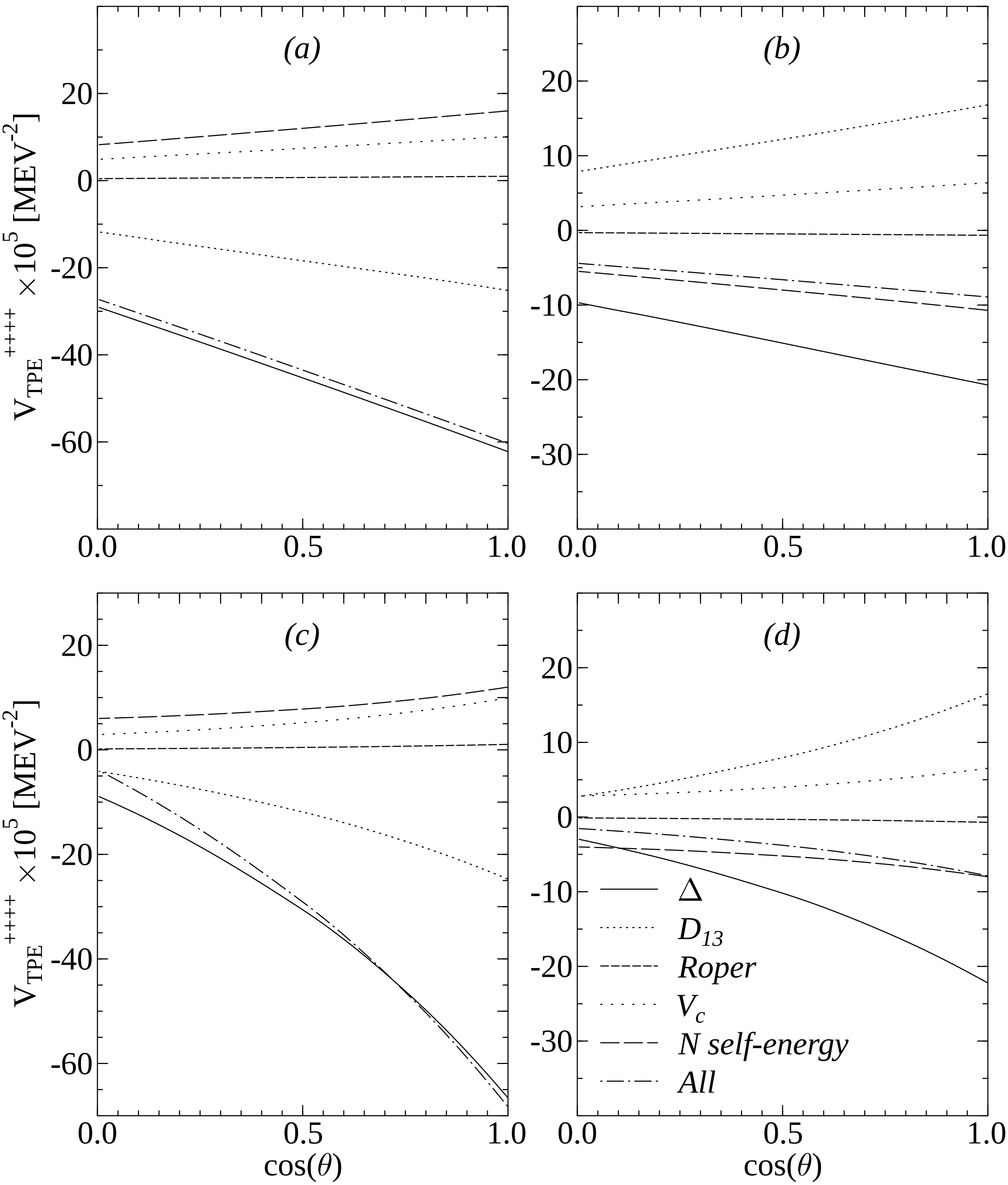






$V_{TPE}{}^{++-+}$ (I=0)
$V_{TPE}{}_{++--}$ (I=0)
$V_{TPE}{}^{++-+}$ (I=1)
$V_{TPE}{}_{++--}$ (I=1)
meson-exchange
$V_{TPE} \times 10^5$ [MEV^{-2}]
P=60 MeV/c
P=240 MeV/c
cos(θ)
cos(θ)
80
60
40
20
0
-20
-40
-60
-80
0.0
0.5
1.0



(a)
(b)
(c)
(d)
$V_{TPE}^{++++} \times 10^5$ [MEV^{-2}]
cos(θ)
Δ
D_{13}
Roper
V_c
N self-energy
All